EE 330
Lecture 23

Small Signal Analysis (an introduction)

Will show that single transistor amplifiers can have a large voltage gain !

Fall 2024 Exam Schedule

Exam 1	Friday	Sept 27
Exam 2	Friday	October 25
Exam 3	Friday	Nov 22
Final Exam	Monday	Dec 16 12:00 - 2:00 PM

Operating Point of Electronic Circuits

Often interested in circuits where a small signal input is to be amplified (e.g. V_M in previous slide is small)

The electrical port variables where the small signals goes to 0 are termed the Operating Points, the Bias Points, the Quiescent Points, or simply the Q-Points

By setting the small signal inputs to 0, it means replacing small voltage inputs with short circuits and small current inputs with open circuits

When analyzing small-signal amplifiers, it is necessary to obtain the Q-point

When designing small-signal amplifiers, establishing of the desired Q-point is termed "biasing"

- Capacitors become open circuits (and inductors short circuits) when determining Q-points
- Simplified dc models of the MOSFET (saturation region) or BJT (forward active region) are usually adequate for determining the Q-point in practical amplifier circuits
- DC voltage and current sources remain when determining Q-points
- Small-signal voltage and current sources are set to 0 when determining Q-points

Amplification with Transistors

From Wikipedia: (Oct. 2022)

An **amplifier**, **electronic amplifier** or (informally) **amp** is an electronic device that can increase the power of a signal (a time-varying voltage or current).

What is the "power" of a signal?

Can an amplifier make decisions?

Does Wikipedia have such a basic concept right?

Amplification with Transistors

From Wikipedia: (October 2023)

An **amplifier**, **electronic amplifier** or (informally) **amp** is an electronic device that can increase the magnitude of a signal (a time-varying voltage or current).

It is a two-port electronic circuit that uses electric power from a power supply to increase the amplitude (magnitude of the voltage or current) of a signal applied to its input terminals, producing a proportionally greater amplitude signal at its output. The amount of amplification provided by an amplifier is measured by its gain: the ratio of output voltage, current, or power to input. An amplifier is defined as a circuit that has a power gain greater than one.[2][3][4]

Even more self-inconsistent definition !

We have had “amplifiers” for over 100 years – will we ever have a consensus on what an amplifier is?

Dependent Sources

What is a dependent source?

Did you ever see one in the EE 201 Course?

Did you have them in your parts kit for EE 201?

Did you ever see one in the EE 201 Laboratory?

Did you ever see one in the EE 230 Laboratory?

Can you buy one from Digi Key?

Digi-Key ELECTRONICS

World's Largest Selection

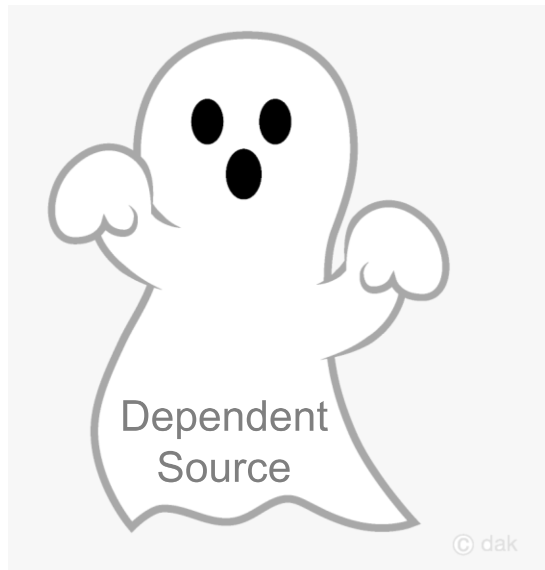
of Electronic Components Available for Immediate Shipment!®

Digi-Key has over 9 million different parts from over 1000 suppliers !

Dependent Sources

What is a dependent source?

Will you suddenly find dependent sources after you graduate ?

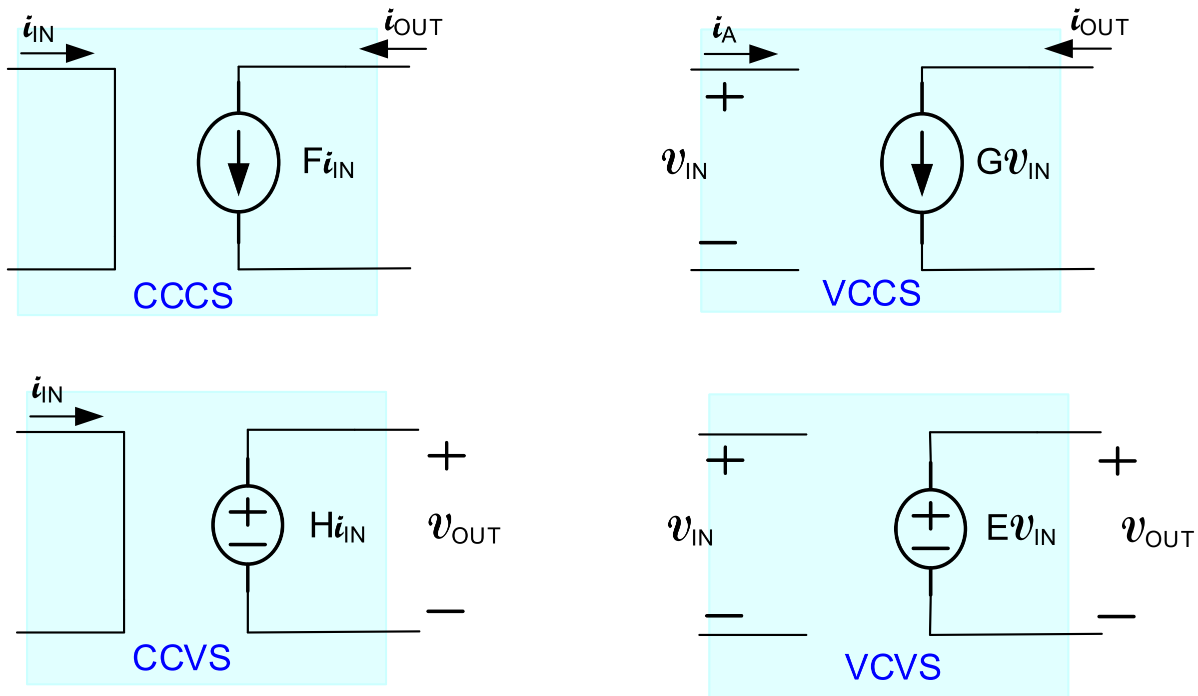


Do dependent sources really exist ?

Why do we place so much emphasis on dependent sources in EE 201?

Dependent Sources

What is a dependent source?

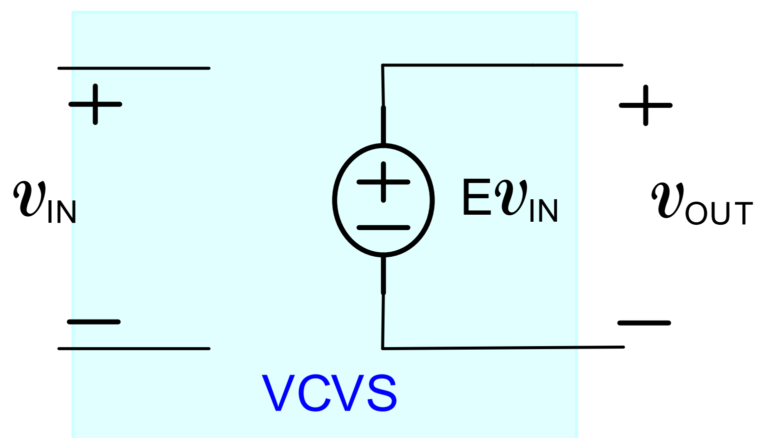


The four basic dependent sources !

Two-port networks with infinite or zero input and output impedances

Dependent Sources

Observe, as an example,



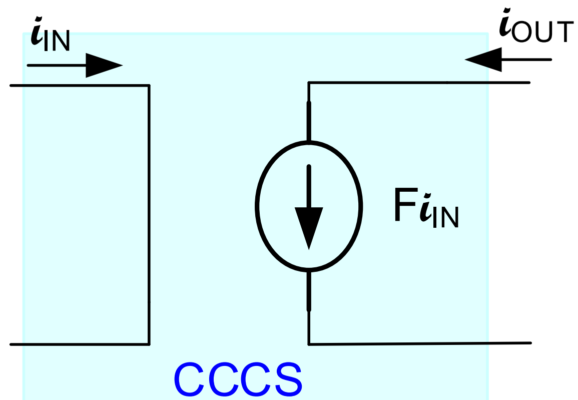
$$V_{OUT} = EV_{IN}$$

$$R_{IN}=\infty \quad R_{OUT}=0$$

Does this have anything in common with a Voltage Amplifier?

Dependent Sources

Observe, as an example,



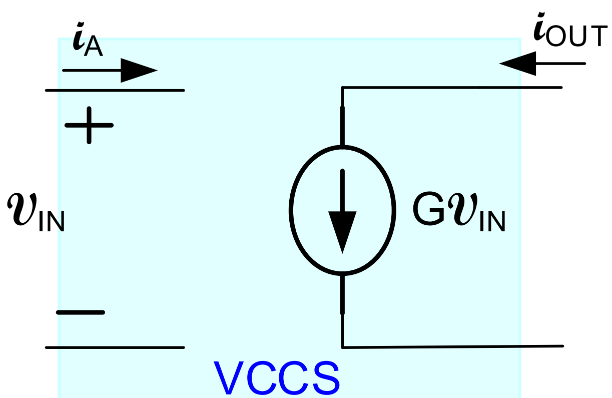
$$I_{OUT} = FI_{IN}$$

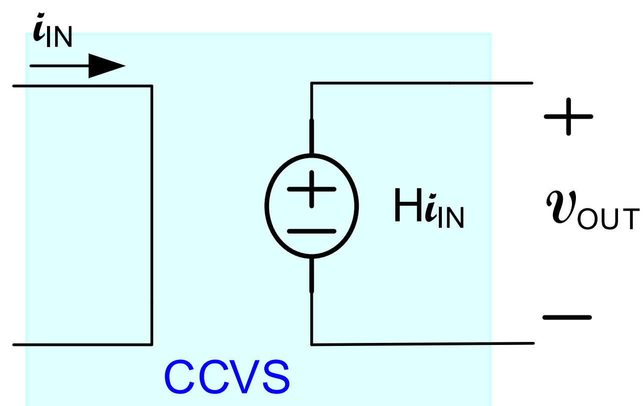
$$R_{IN}=0 \quad R_{OUT}=\infty$$

Does this have anything in common with a Current Amplifier?

Dependent Sources

Observe, as an example,





$I_{OUT} = GV_{IN}$

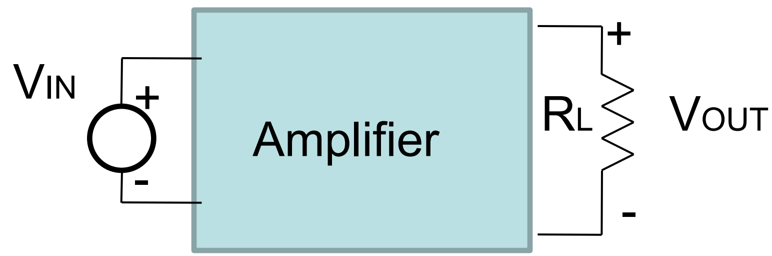
$R_{IN}=\infty \quad R_{OUT}=\infty$

$V_{OUT} = HI_{IN}$

$R_{IN}=0 \quad R_{OUT}=0$

What about these dependent sources?

Amplification with Transistors



Often the voltage gain of an amplifier is larger than 1

$$V_{OUT} = A_V V_{IN}$$

Often (but not always) the power dissipated by R_L is larger than the power supplied by V_{IN}

An amplifier can be thought of as a dependent source that was discussed in EE 201

Input and output variables can be either V or I or mixed

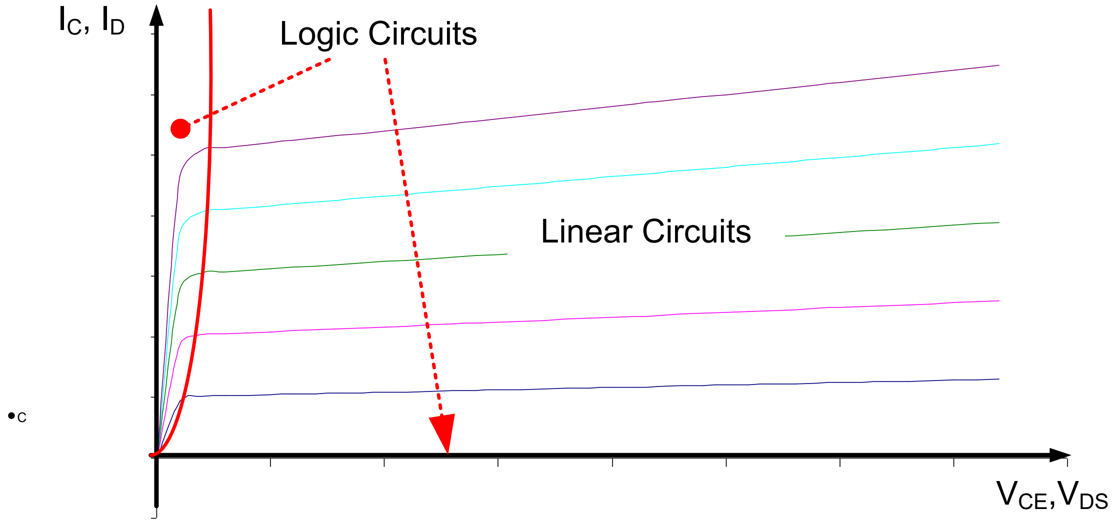
Amplifier

From Wikipedia: (March 2022)

An **amplifier**, **electronic amplifier** or (informally) **amp** is an electronic device that can increase the power of a signal (a time-varying voltage or current).

An amplifier is another name for any for the four basic dependent sources that are discussed in basic circuits textbooks.

Applications of Devices as Amplifiers

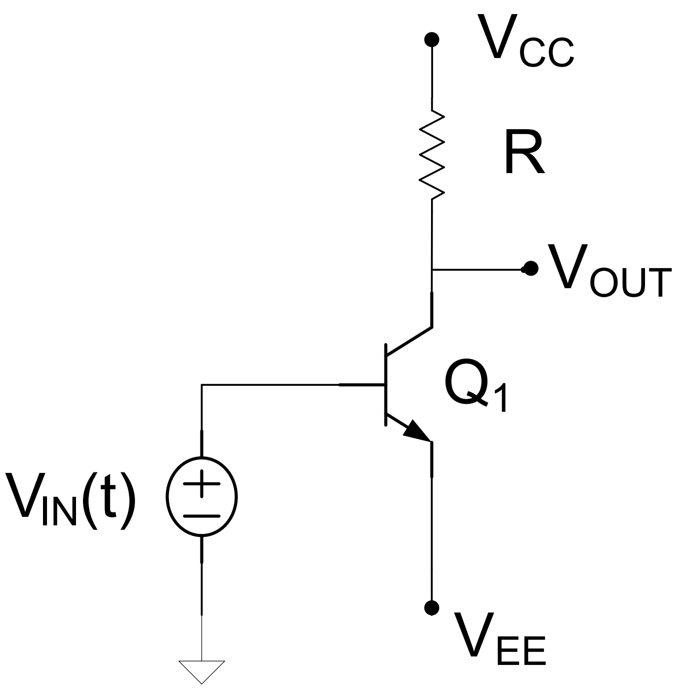


Typical Regions of Operation by Circuit Function

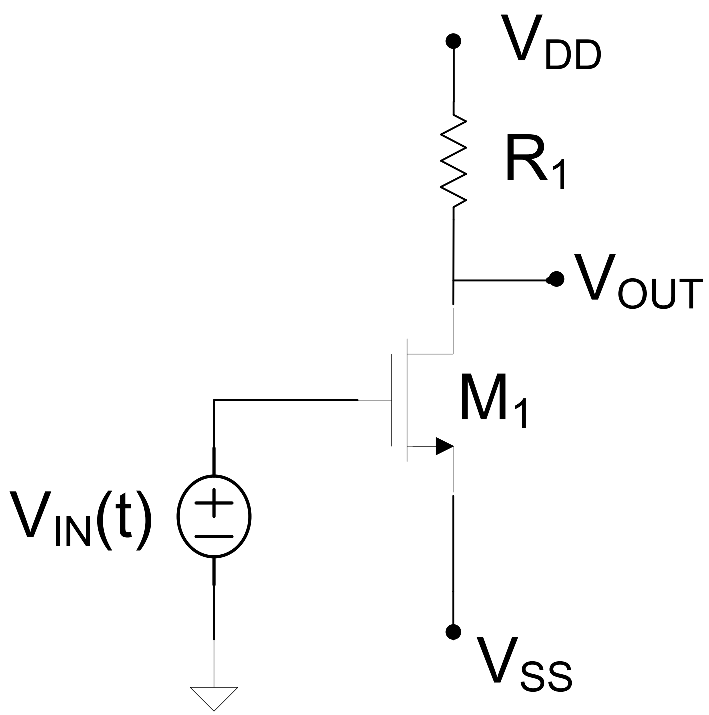
	MOS	Bipolar
Logic Circuits	Triode and Cutoff	Saturation and Cutoff
Linear Circuits	Saturation	Forward Active

Consider the following MOSFET and BJT Circuits

BJT



MOSFET



Assume BJT operating in FA region, MOSFET operating in Saturation

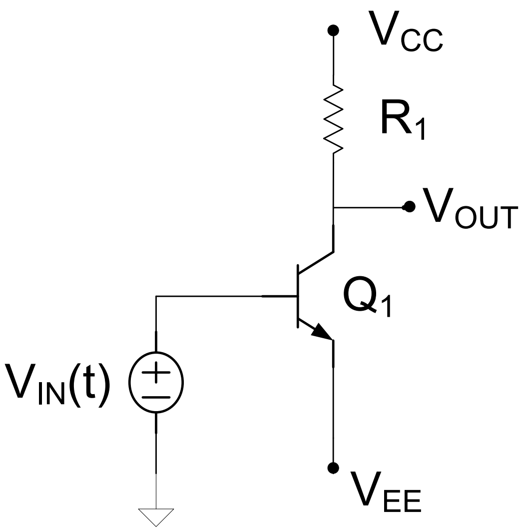
Assume same quiescent output voltage and same resistor ($R=R_1$)

Note architecture is same for BJT and MOSFET circuits

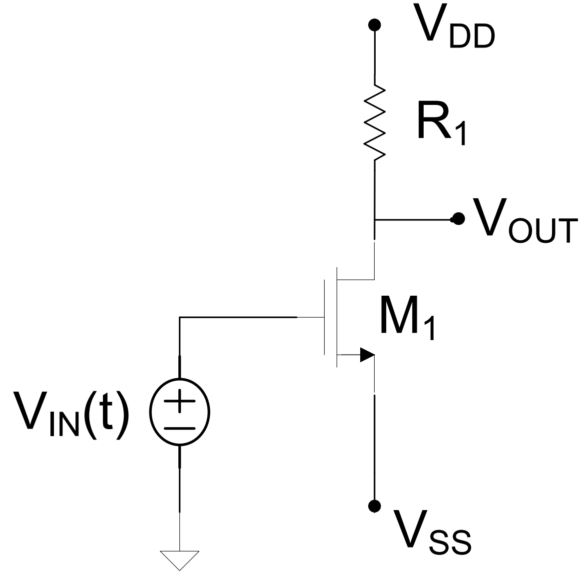
One of the most widely used amplifier architectures

Consider the following MOSFET and BJT Circuits

BJT



MOSFET



- MOS and BJT Architectures often Identical
- Circuits are Highly Nonlinear
- Nonlinear Analysis Methods Must be used to analyze these and almost any other nonlinear circuit

Methods of Analysis of Nonlinear Circuits

KCL and KVL apply to both linear and nonlinear circuits

Superposition, voltage divider and current divider equations, Thevenin and Norton equivalence apply only to linear circuits!

Some other analysis techniques that have been developed may apply only to linear circuits as well

Methods of Analysis of Nonlinear Circuits

Will consider three different analysis requirements and techniques for some particularly common classes of nonlinear circuits

1. Circuits with continuously differential devices

   Interested in obtaining transfer characteristics of these circuits or outputs for given input signals

2. Circuits with piecewise continuous devices

   Interested in obtaining transfer characteristics of these circuits or outputs for a given input signals

3. Circuits with small-signal inputs that vary around some operating point

   Interested in obtaining relationship between small-signal inputs and the corresponding small-signal outputs. Will assume these circuits operate linearly in some suitably small region around the operating point

Other types of nonlinearities may exist and other types of analysis may be required but we will not attempt to categorize these scenarios in this course

1. Nonlinear circuits with continuously differential devices

Analysis Strategy:

Use KVL and KCL for analysis

Represent nonlinear models for devices either mathematically or graphically

Solve the resultant set of nonlinear and linear equations (often differential equations) for the variables of interest

2. Circuits with piecewise continuous devices

e.g. $$f(x)=\begin{cases} f_1(x) & x<x_1 \quad \text{region 1} \\ f_2(x) & x>x_1 \quad \text{region 2} \end{cases}$$

Analysis Strategy:

Guess region of operation

Solve resultant circuit using the previous method

Verify region of operation is valid

Repeat the previous 3 steps as often as necessary until region of operation is verified

- It helps to guess right the first time but a wrong guess will not result in an incorrect solution because a wrong guess can not be verified

- Piecewise models generally result in a simplification of the analysis of nonlinear circuits

3. Circuits with small-signal inputs that vary around some operating point

Interested in obtaining relationship between small-signal inputs and the corresponding small-signal outputs. Will assume these circuits operate linearly in some suitably small region around the operating point

Analysis Strategy:

Use methods from previous two class of nonlinear circuits

More Practical Analysis Strategy:

Determine the operating point (using method 1 or 2 discussed above after all small signal independent inputs are set to 0)

Develop small signal (linear) model for all devices in the region of interest (around the operating point or "Q-point")

Create small signal equivalent circuit by replacing all devices with small-signal equivalent

Solve the resultant small-signal (linear) circuit

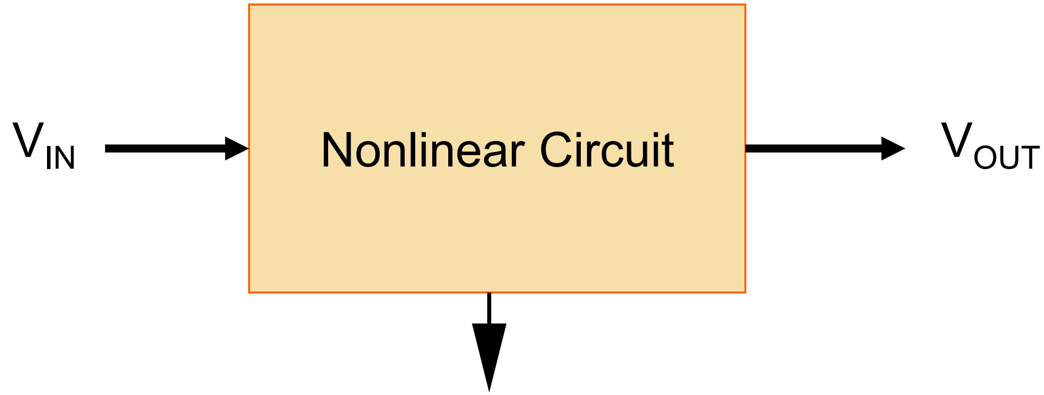
Can use KCL, DVL, and other linear analysis tools such as superposition, voltage and current divider equations, Thevenin and Norton equivalence

Determine boundary of region where small signal analysis is valid

Small signal operation of nonlinear circuits

$V_{IN}=V_M\sin\omega t$

V_M is small



If V_M is sufficiently small, then any nonlinear circuit operating at a region where there are no abrupt nonlinearities will have a nearly sinusoidal output and the variance of the magnitude of this output with V_M will be nearly linear (could be viewed as "locally linear")

This is termed the "small signal" operation of the nonlinear circuit

When operating with "small signals", the nonlinear circuit performs linearly with respect to these small signals thus other properties of linear networks such as superposition apply provided the sum of all superimposed signals remains sufficiently small

Other types of "small signals", e.g. square waves, triangular waves, or even arbitrary waveforms often are used as inputs as well but the performance of the nonlinear network also behaves linearly for these inputs

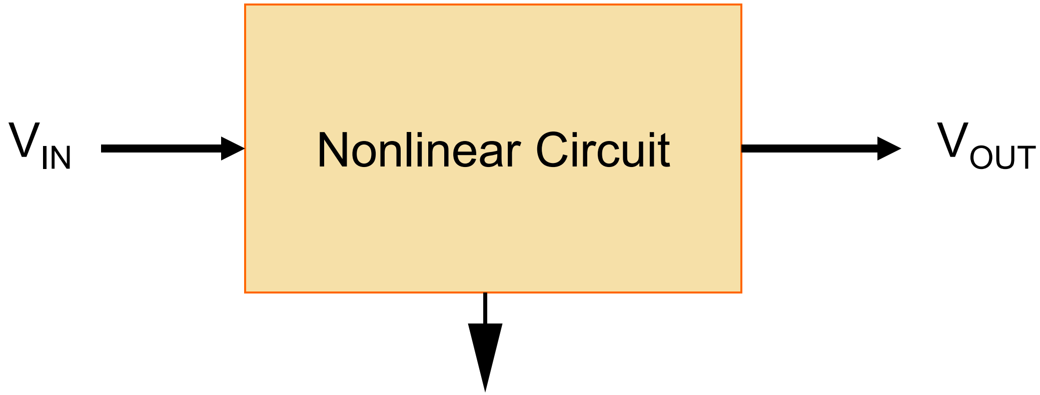
Many useful electronic systems require the processing of these small signals

Practical methods of analyzing and designing circuits that operate with small signal inputs are really important

Small signal operation of nonlinear circuits

$V_{IN} = V_M \sin\omega t$

V_M is small



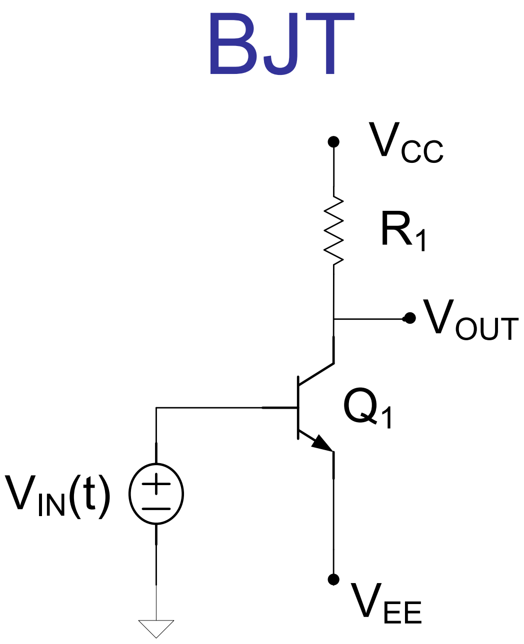
Practical methods of analyzing and designing circuits that operate with small signal inputs are really important

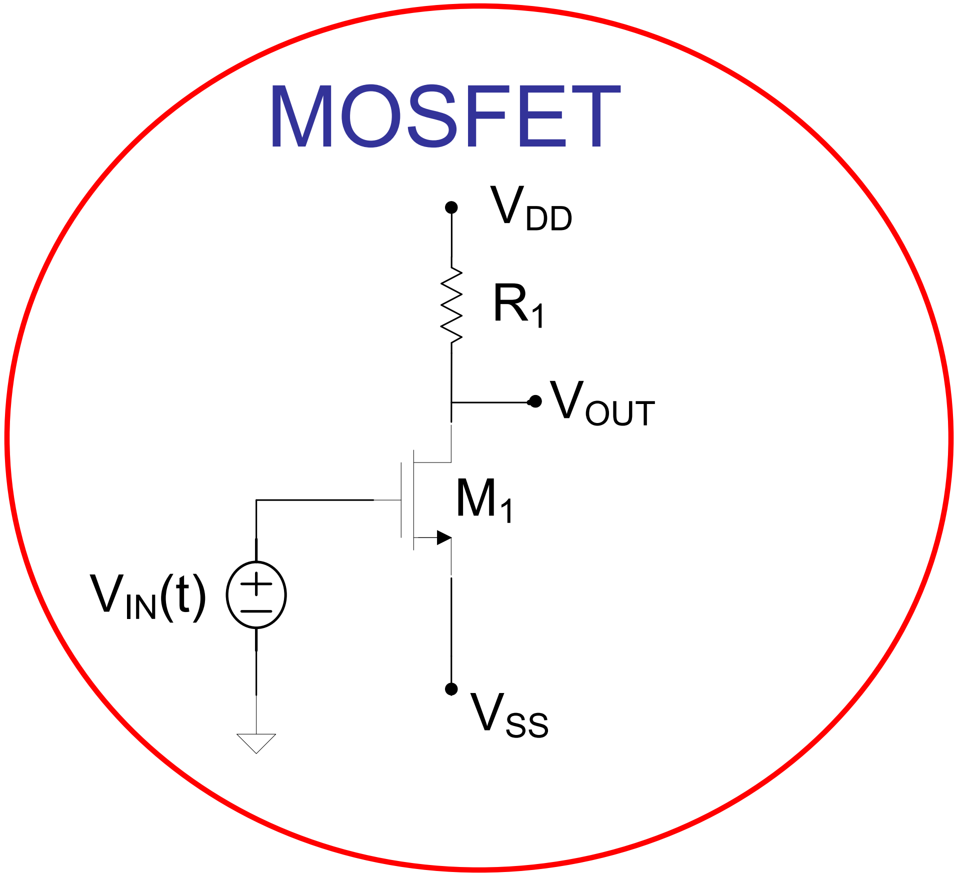
Two key questions:

How small must the input signals be to obtain locally-linear operation of a nonlinear circuit?

How can these locally-linear (alt small signal) circuits be analyzed and designed?

Consider the following MOSFET and BJT Circuits





One of the most widely used amplifier architectures

Small signal operation of nonlinear circuits

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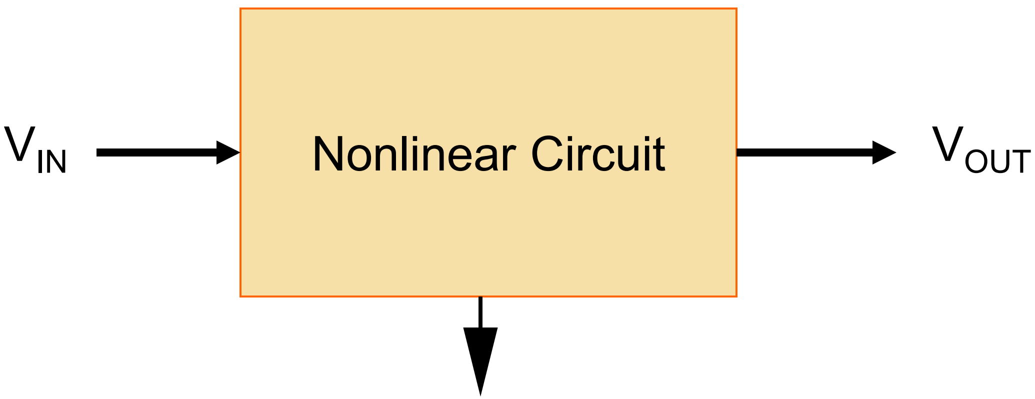
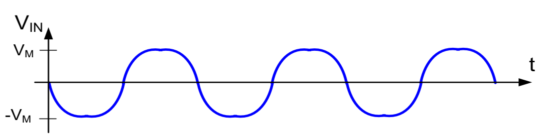
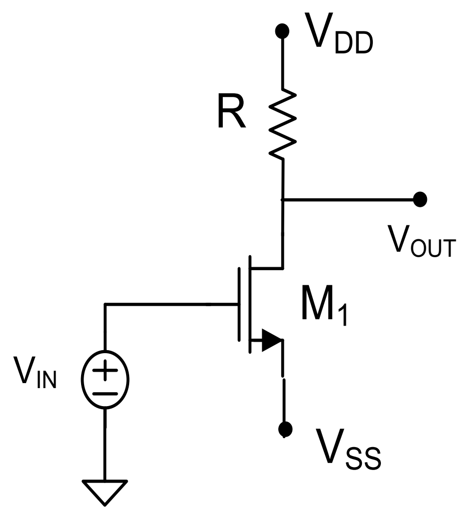
V_M is small



Example of circuit that is widely used in locally-linear mode of operation



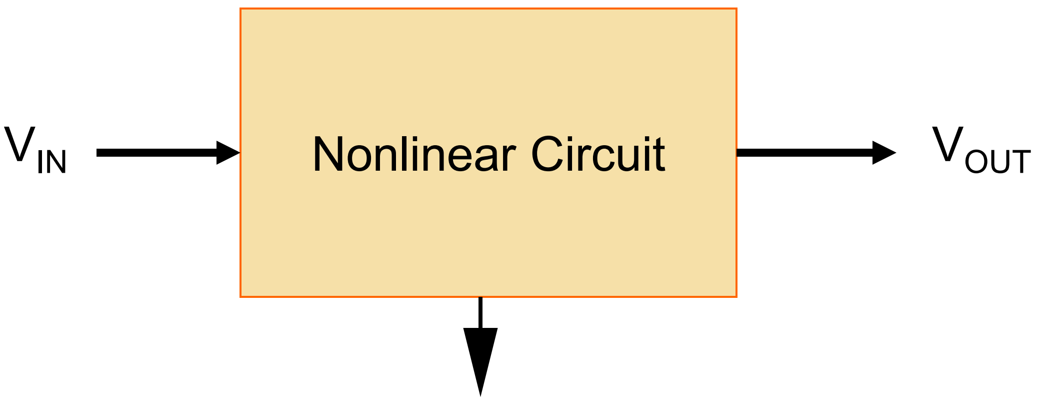


Two methods of analyzing locally-linear circuits will be considered, one of these is by far more practical

Small signal operation of nonlinear circuits

$V_{IN}=V_M \sin\omega t$

V_M is small

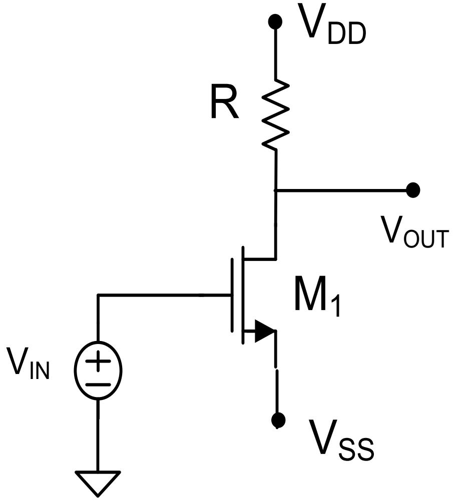


Two methods of analyzing locally-linear circuits for small-signal excitaions will be considered, one of these is by far the most practical

1. Analysis using nonlinear models

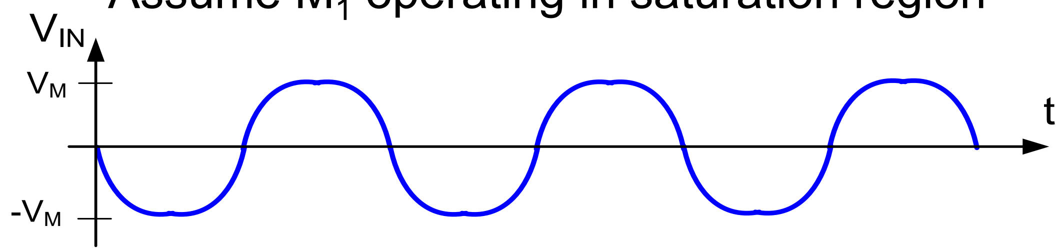
2. Small signal analysis using locally-linearized models

Small signal analysis using nonlinear models



By selecting appropriate value of V_{SS}, M_1 will operate in the saturation region *(this is biasing)*

Assume M_1 operating in saturation region



$V_{IN}=V_M \sin\omega t$

V_M is small

Termed Load Line

$$V_{OUT}=V_{DD}-I_D R$$

$$I_D=\frac{\mu C_{OX} W}{2L}\left(V_{IN}-V_{SS}-V_T\right)^2$$

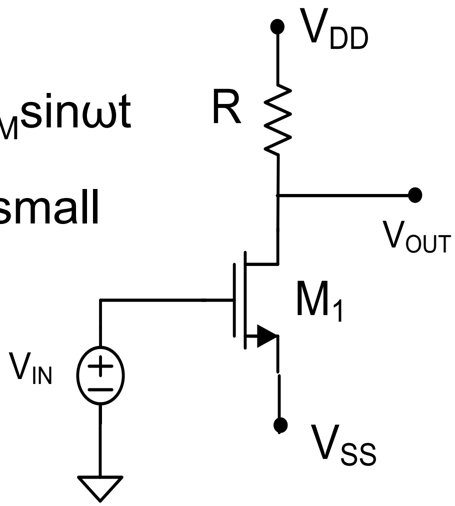
Device Model

$$I_{DQ}=\frac{\mu C_{OX} W}{2L}\left(V_{SS}+V_T\right)^2$$

$$V_{OUT}=V_{DD}-\frac{\mu C_{OX} W}{2L}\left(V_{IN}-V_{SS}-V_T\right)^2 R$$

$$V_{OUT}=V_{DD}-\frac{\mu C_{OX} W}{2L}\left(V_M \sin\omega t-\left[V_{SS}+V_T\right]\right)^2 R$$

Small signal analysis example



$V_{IN}=V_M \sin\omega t$

V_M is small

$$V_{OUT} = V_{DD} - \frac{\mu C_{OX} W}{2L}\left(V_M \sin\omega t - [V_{SS} + V_T]\right)^2 R$$

Note relationship between input and output not linear !

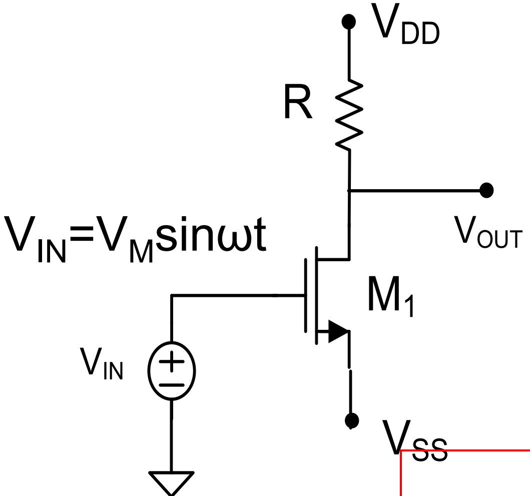
$$V_{OUT} = V_{DD} - \frac{\mu C_{OX} W}{2L}[V_{SS} + V_T]^2\left(1 - \frac{V_M \sin\omega t}{[V_{SS} + V_T]}\right)^2 R$$

Recall that if x is small $(1+x)^2 \cong 1+2x$

$$V_{OUT} \cong V_{DD} - \frac{\mu C_{OX} W}{2L}[V_{SS} + V_T]^2\left(1 - \frac{2V_M \sin\omega t}{[V_{SS} + V_T]}\right) R$$

$$V_{OUT} \cong \left\{V_{DD} - \frac{\mu C_{OX} W}{2L}[V_{SS} + V_T]^2 R\right\} + \left\{\frac{\mu C_{OX} W}{L}[V_{SS} + V_T] R\right\} V_M \sin\omega t$$

Small signal analysis example



$$V_{OUT} \cong \left\{V_{DD} - \frac{\mu C_{OX} W}{2L}\left[V_{SS}+V_T\right]^2 R\right\} + \left\{\frac{\mu C_{OX} W}{L}\left[V_{SS}+V_T\right] R\right\} V_M \sin\omega t$$

Quiescent Output *ss Voltage Gain*

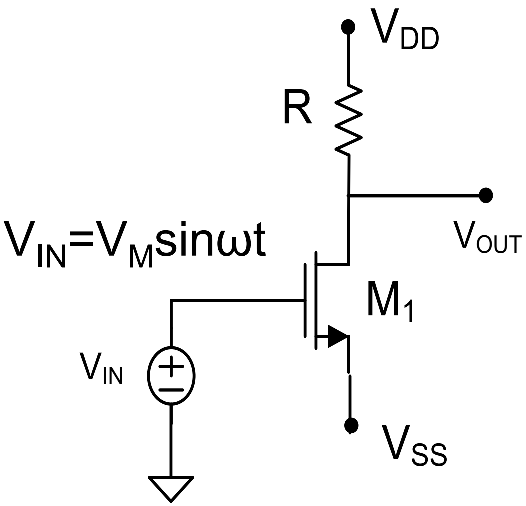
$$A_v = \frac{\mu C_{OX} W}{L}\left[V_{SS}+V_T\right] R$$

$$V_{OUTQ} = \left\{V_{DD} - \frac{\mu C_{OX} W}{2L}\left[V_{SS}+V_T\right]^2 R\right\}$$

$$V_{OUT} \cong V_{OUTQ} + A_V V_M \sin\omega t$$

Note the ss voltage gain is negative since $V_{SS}+V_T<0$!

Small signal analysis example



$$V_{OUT} \cong V_{OUTQ} + A_V V_M \sin\omega t$$

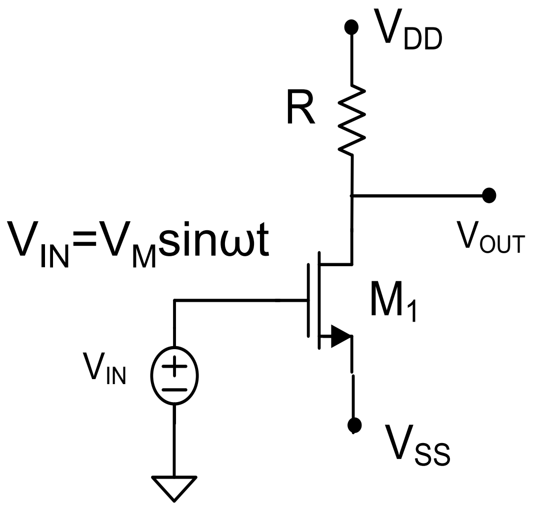
$$A_v = \frac{\mu C_{OX} W}{L}\left[V_{SS} + V_T\right] R$$

$$V_{OUTQ} = \left\{ V_{DD} - \frac{\mu C_{OX} W}{2L}\left[V_{SS} + V_T\right]^2 R \right\}$$

But – this expression gives little insight into how large the gain is !

And the analysis for even this very simple circuit was messy!

Small signal analysis example



$$V_{OUT} \cong V_{OUTQ} + A_V V_M \sin\omega t$$

$$A_v = \frac{\mu C_{ox} W}{L}\left[V_{ss} + V_T\right] R$$

As an example, if W=20u, L=1u, R=50K, V_{DD}=3V, V_{TH}=0.5V, uC_{OX}=100uA/V^2, V_{SS}=-.7V,

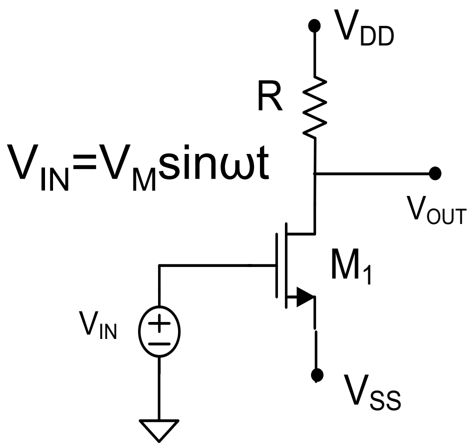
Substituting into the expressions for A_V, V_{OUTQ} and I_{DQ}, obtain

A_V = -20, I_{DQ}=40uA, V_{OUTQ}=1V

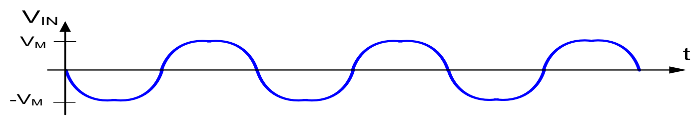
This circuit has Voltage Gain ! !

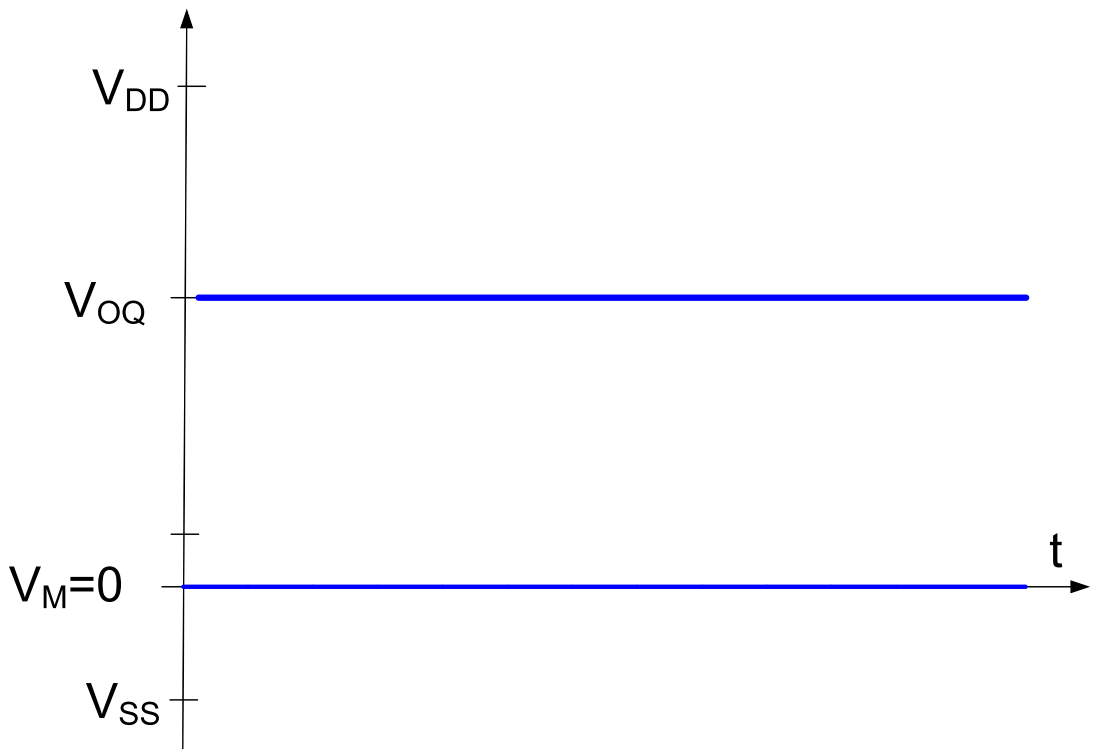
And if A_V >>1, this circuit has a lot of gain!

Small signal analysis example

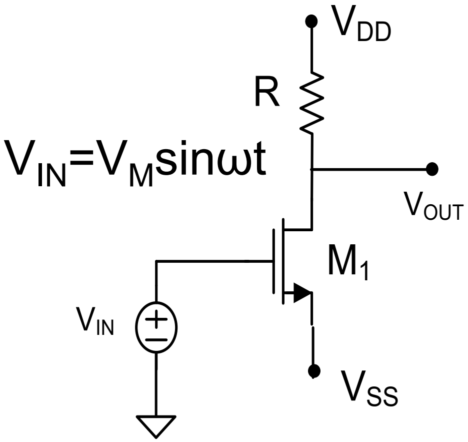


$$V_{OUT} \cong V_{OUTQ} + A_V V_M \sin\omega t$$



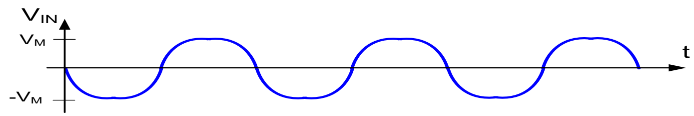


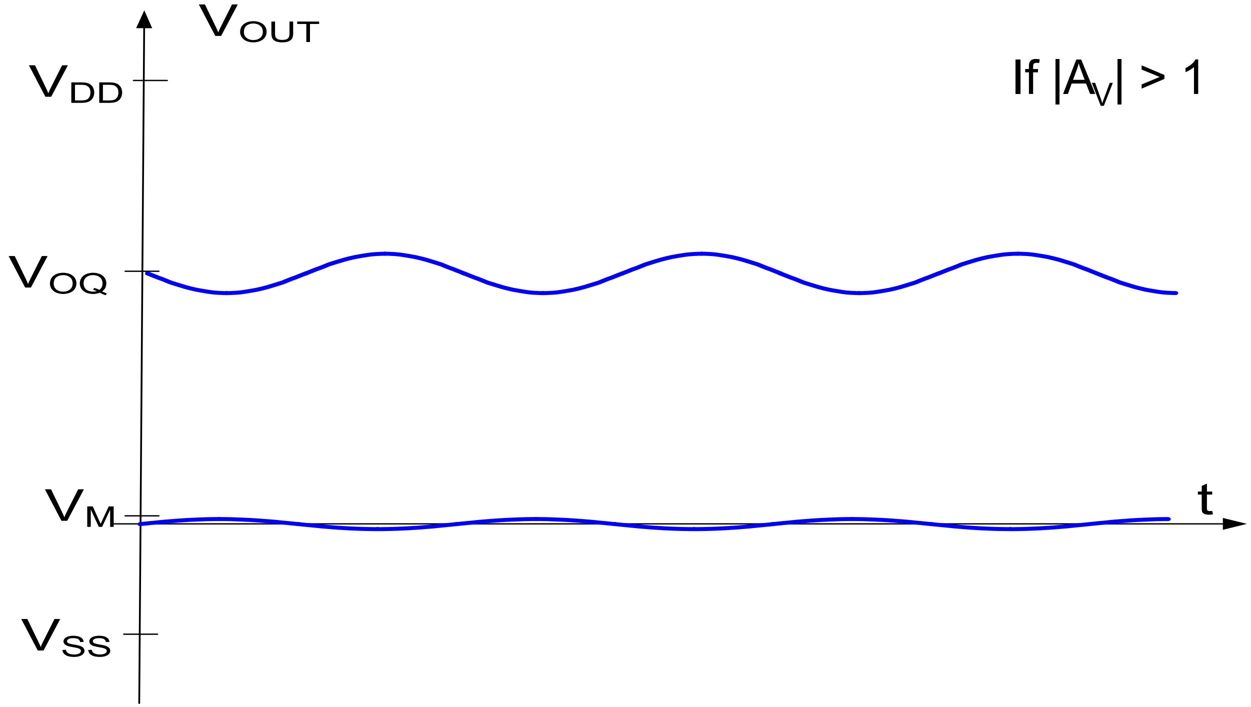
Small signal analysis example



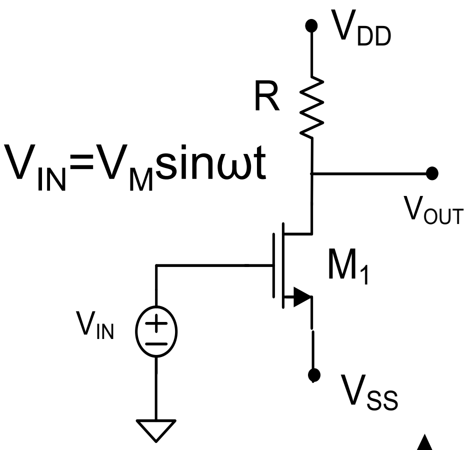
$$V_{OUT} \cong V_{OUTQ} + A_V V_M \sin\omega t$$

$$A_v = \frac{\mu C_{OX} W}{L}\left[V_{SS} + V_T\right] R$$



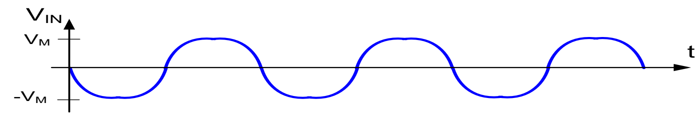


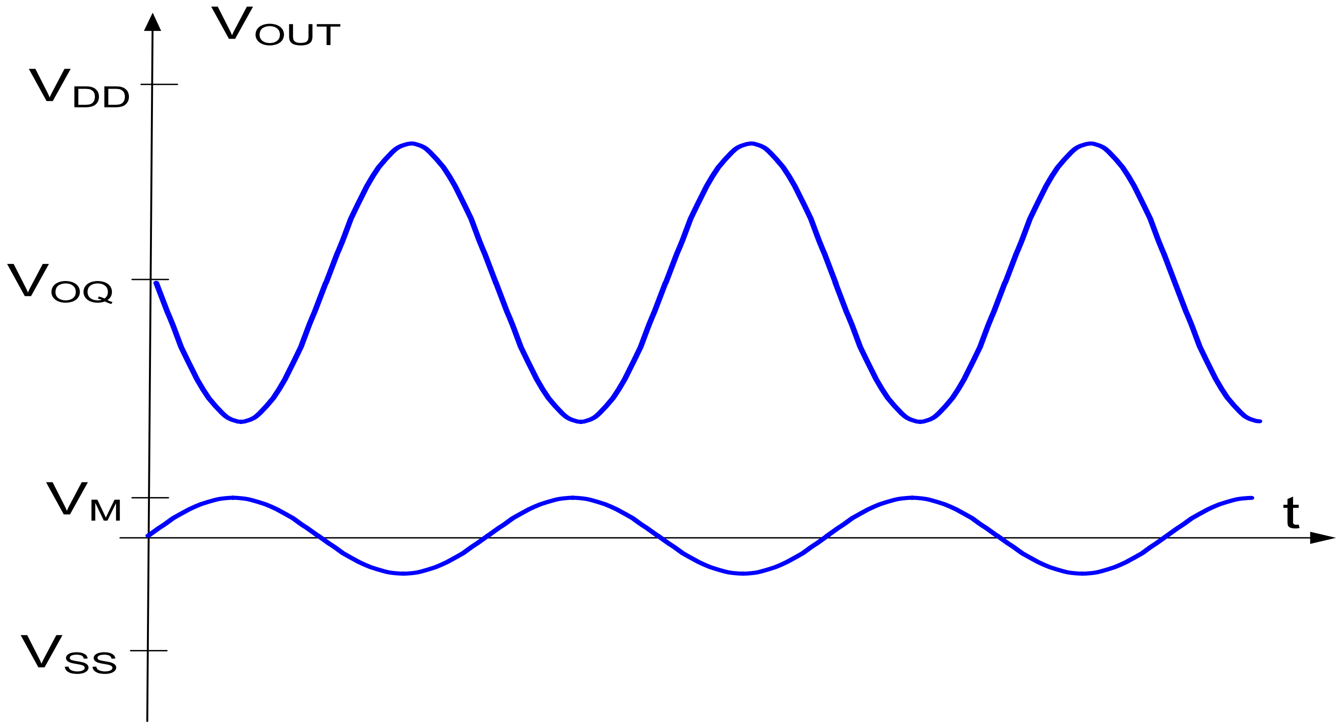
Small signal analysis example



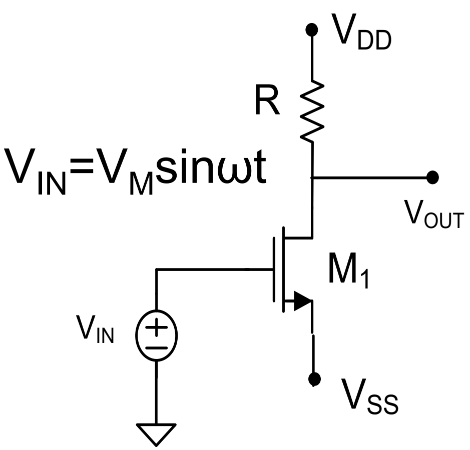
$$V_{OUT} \cong V_{OUTQ} + A_V V_M \sin\omega t$$

$$A_v = \frac{\mu C_{OX} W}{L}\left[V_{SS} + V_T\right] R$$

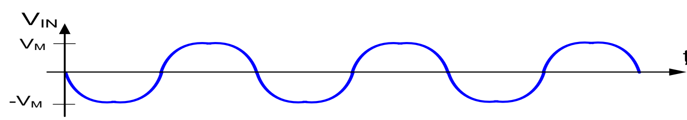




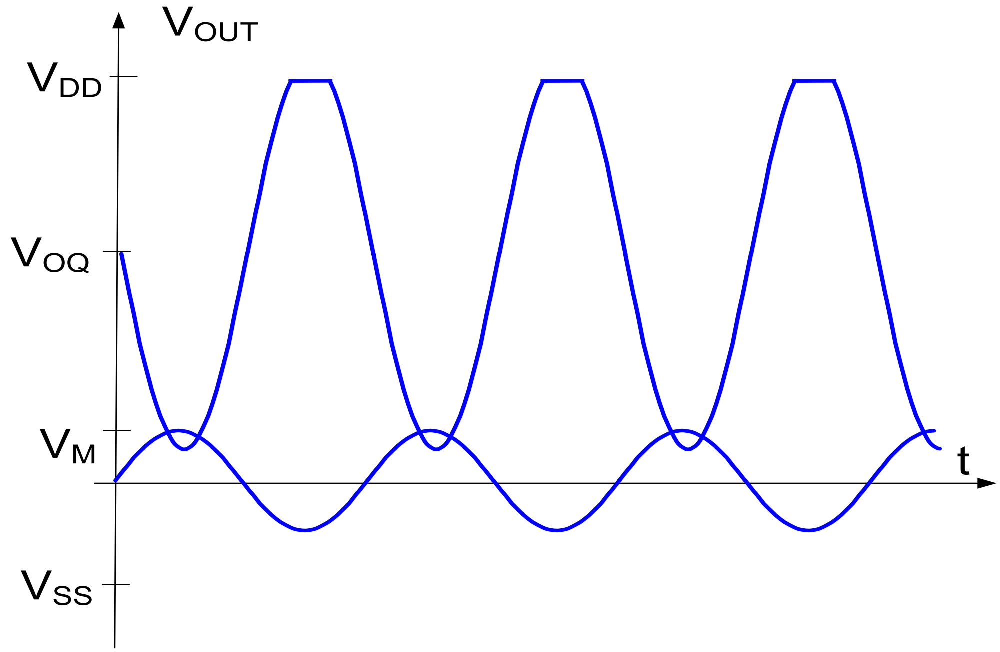
Small signal analysis example



$$V_{OUT} \cong V_{OUTQ} + A_V V_M \sin\omega t$$



$$A_v = \frac{\mu C_{OX} W}{L}\left[V_{SS} + V_T\right] R$$

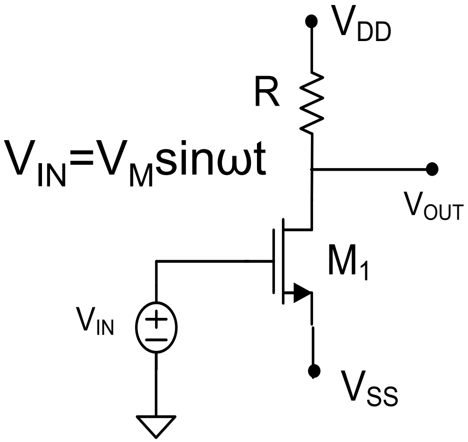


Serious Distortion occurs if signal is too large or Q-point non-optimal

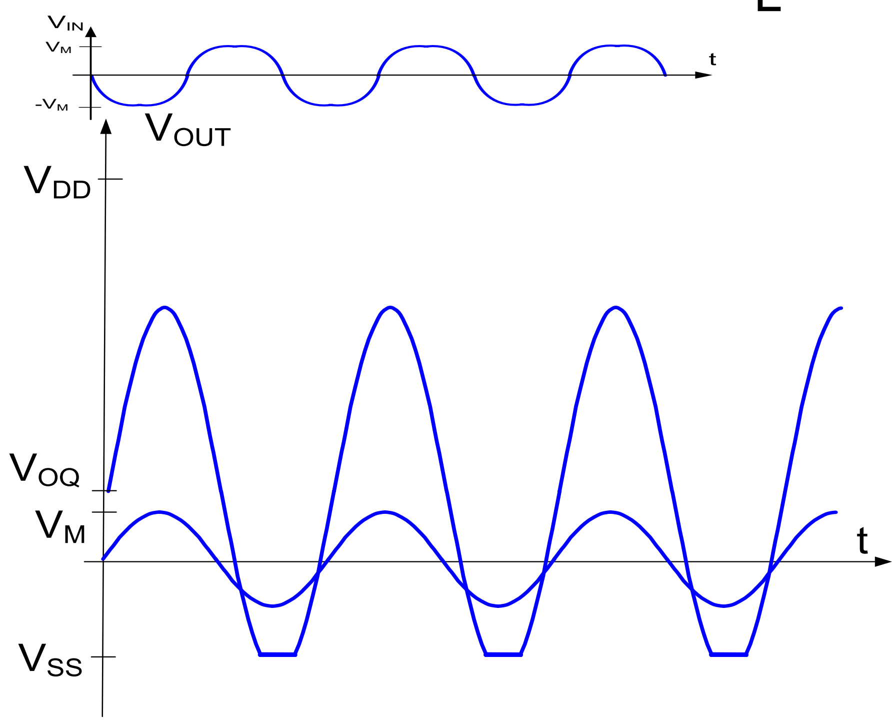
Here “clipping” occurs for high V_{OUT}

Small signal analysis example

$$V_{OUT} \cong V_{OUTQ} + A_V V_M \sin\omega t$$



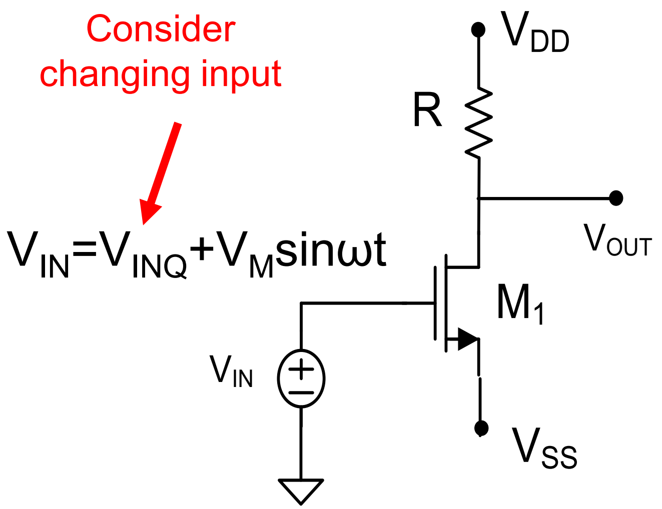
$$A_v = \frac{\mu C_{ox} W}{L}\left[V_{SS} + V_T\right] R$$



Serious Distortion occurs if signal is too large or Q-point non-optimal

Here “clipping” occurs for low V_{OUT}

Small signal analysis example



Assume M_1 operating in saturation region
When $V_{IN}=V_{INQ}$, the Q-point solution:

$$I_{DQ}=\frac{\mu C_{OX}W}{2L}\left(V_{INQ}-V_{SS}-V_T\right)^2$$

$$V_{OUTQ}=V_{DD}-\frac{\mu C_{OX}W}{2L}\left[V_{INQ}-V_{SS}-V_T\right]^2R$$

Near the Q-point, small signals have linear relationship:

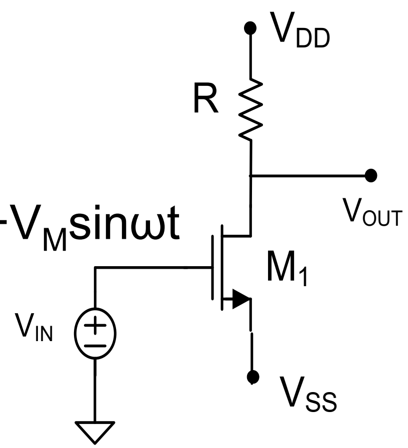
$$V_{OUTsmall}=V_{OUT}-V_{OUTQ}\cong A_V\cdot(V_{IN}-V_{INQ})=A_VV_M\sin\omega t$$

$$V_{OUTsmall}\cong A_VV_{INsmall}$$

Following same analysis approach used when $V_{INQ}=0$, obtain

$$A_V\cong-\frac{\mu C_{OX}W}{L}\left[V_{INQ}-V_{SS}-V_T\right]R$$

Small signal analysis example



$V_{IN}=V_{INQ}+V_M\sin\omega t$

$$V_{OUT} \cong V_{OUTQ} + A_V V_M \sin\omega t$$

$$A_V \cong -\frac{\mu C_{OX} W}{L}\left[V_{INQ}-V_{SS}-V_T\right]R$$

$$V_{OUTQ}=V_{DD}-\frac{\mu C_{OX} W}{2L}\left[V_{INQ}-V_{SS}-V_T\right]^2 R$$

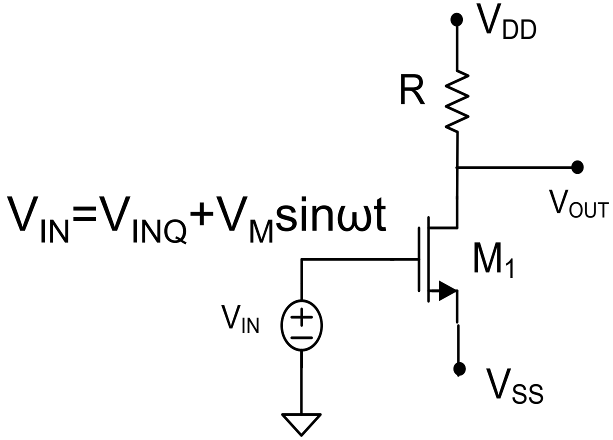
But – this expression gives little insight into how large the gain is !

Can the gain be made arbitrarily large by simply making R large?

Observe increasing R with W,L, and V_{SS} fixed will change Q-point and may cause M_1 to leave saturation

Difficult to answer this question with the information provided !

Small signal analysis example



$$V_{OUT} \cong V_{OUTQ} + A_V V_M \sin\omega t$$

$$A_V \cong -\frac{\mu C_{OX} W}{L}\left[V_{INQ} - V_{SS} - V_T\right]R$$

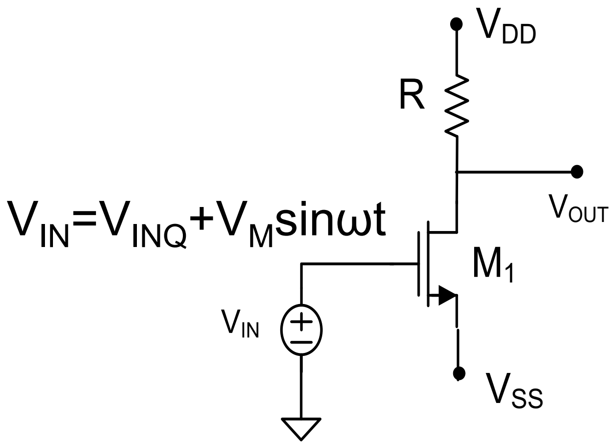
But recall:

$$I_{DQ} = \frac{\mu C_{OX} W}{2L}\left(V_{INQ} - V_{SS} - V_T\right)^2$$

Thus, substituting from the expression for I_{DQ} we obtain

$$A_v = -\frac{2I_{DQ}R}{\left[V_{INQ} - V_{SS} - V_T\right]} = -\frac{2I_{DQ}R}{\left[V_{GSQ} - V_T\right]}$$

Small signal analysis example



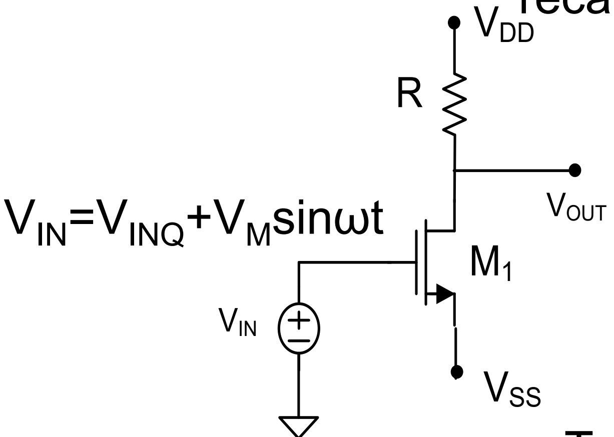
$$A_v = -\frac{2I_{DQ}R}{[V_{GSQ}-V_T]}$$

- Small signal voltage gain is twice the Quiescent voltage across R divided by $V_{GSQ} - V_T$
- Making $I_{DQ}R$ too big or too small will limit signal swing (cause M_1 to leave saturation region)
- Can make $|A_V|$ large by making $V_{GSQ} - V_T$ small
- A_V increases proportionally to the power dissipation (from supply) for fixed V_{GSQ}

- This analysis which required linearization of a nonlinear output voltage is quite tedious.
- This approach becomes unwieldy for even slightly more complicated circuits
- A much easier approach based upon the development of small signal models will provide the same results, provide more insight into both analysis and design, and result in a dramatic reduction in computational requirements

Small signal analysis example

(Consider what was neglected in the previous analysis)



recall

$$V_{OUT} \cong V_{OUTQ} + A_V V_M \sin\omega t$$

However, there are invariably small errors in this analis

$$V_{OUT} = V_{OUTQ} + A_V V_M \sin\omega t + \varepsilon(t)$$

To see the effects of the approximations consider again

$$V_{OUT} = V_{DD} - \frac{\mu C_{OX} RW}{2L}\left(V_M \sin(\omega t) + \left[V_{GSQ} - V_T\right]\right)^2$$

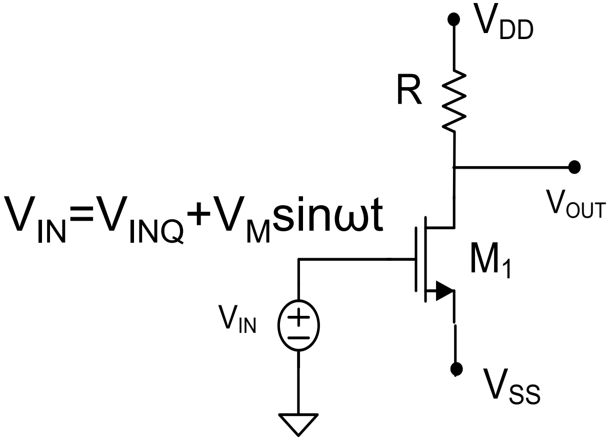
$$V_{OUT} = V_{DD} - \frac{\mu C_{OX} RW}{2L}\left(V_M^2 \sin(\omega t)^2 + 2\left[V_{GSQ} - V_T\right]V_M \sin\omega t + \left[V_{GSQ} - V_T\right]^2\right)$$

$$V_{OUT} = V_{DD} - \frac{\mu C_{OX} RW}{2L}\left(V_M^2\left[\frac{1-\cos 2\omega t}{2}\right] + 2\left[V_{GSQ} - V_T\right]V_M \sin\omega t + \left[V_{GSQ} - V_T\right]^2\right)$$

$$V_{OUT} = \left\{V_{DD} - \frac{\mu C_{OX} RW}{2L}\left(\frac{V_M^2}{2} + \left[V_{GSQ} - V_T\right]^2\right)\right\} - \left\{\left(\frac{\mu C_{OX} W}{L}\left[V_{GST} - V_T\right]R\right)V_M \sin\omega t\right\} + \left\{\left(\frac{\mu C_{OX} RW}{4L} V_M^2\right)\cos 2\omega t\right\}$$

Note presence of second harmonic distortion term which is a major part of ε(t) !

Small signal analysis example



Nonlinear distortion term

$$V_{OUT} \cong V_{OUTQ} + A_V V_M \sin\omega t$$

$$V_{OUT} = V_{OUTQ} + A_V V_M \sin\omega t + \varepsilon(t)$$

$$V_{OUT} = \left\{V_{DD} - \frac{\mu C_{OX} RW}{2L}\left(\frac{V_M^2}{2} + \left[V_{GSQ} - V_T\right]^2\right)\right\} - \left\{\left(\frac{\mu C_{OX} W}{L}\left[V_{GSQ} - V_T\right]R\right) V_M \sin\omega t\right\} + \left\{\left(\frac{\mu C_{OX} RW}{4L} V_M^2\right)\cos 2\omega t\right\}$$

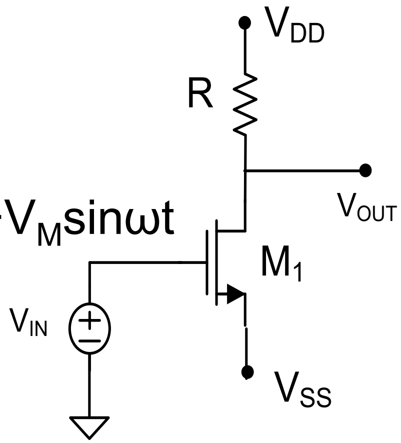
$$V_{OUTDC} = V_{OUTQ} - \frac{\mu C_{OX} RW}{4L} V_M^2$$

$$A_V = -\frac{\mu C_{OX} W}{L}\left[V_{GSQ} - V_T\right]R$$

$$A_2 = \frac{\mu C_{OX} RW}{4L} V_M$$

$$V_{OUT} = V_{OUTDC} + \left\{A_V V_M \sin\omega t\right\} + \left\{A_2 V_M \cos 2\omega t\right\}$$

Small signal analysis example



$V_{IN}=V_{INQ}+V_M \sin\omega t$

Nonlinear distortion term

$$V_{OUT}=V_{OUTDC}+\left\{A_V V_M \sin\omega t\right\}+\left\{A_2 V_M \cos 2\omega t\right\}$$

$$A_V=-\frac{\mu C_{OX} W}{L}\left[V_{GSQ}-V_T\right]R \qquad A_2=\frac{\mu C_{OX} R W}{4L}V_M$$

Total Harmonic Distortion:

Recall, if $x(t)=\sum_{k=0}^{\infty} b_k \sin(k\omega T+\phi_k)$ then $THD=\frac{\sqrt{\sum_{k=2}^{\infty} b_k^2}}{|b_1|}$

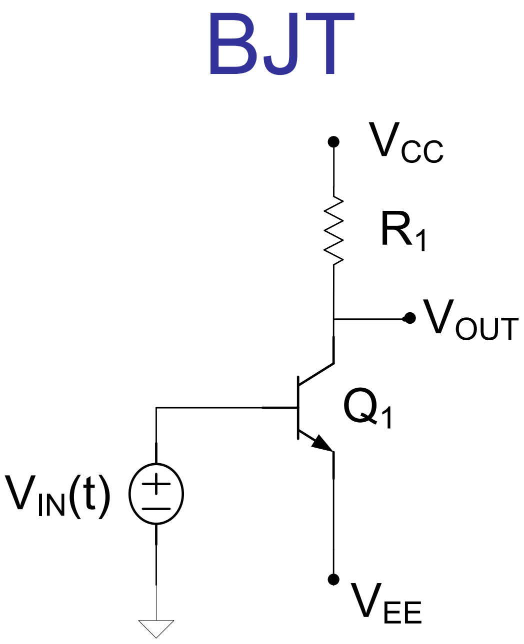
Thus, for this amplifier, as long as M_1 stays in the saturation region

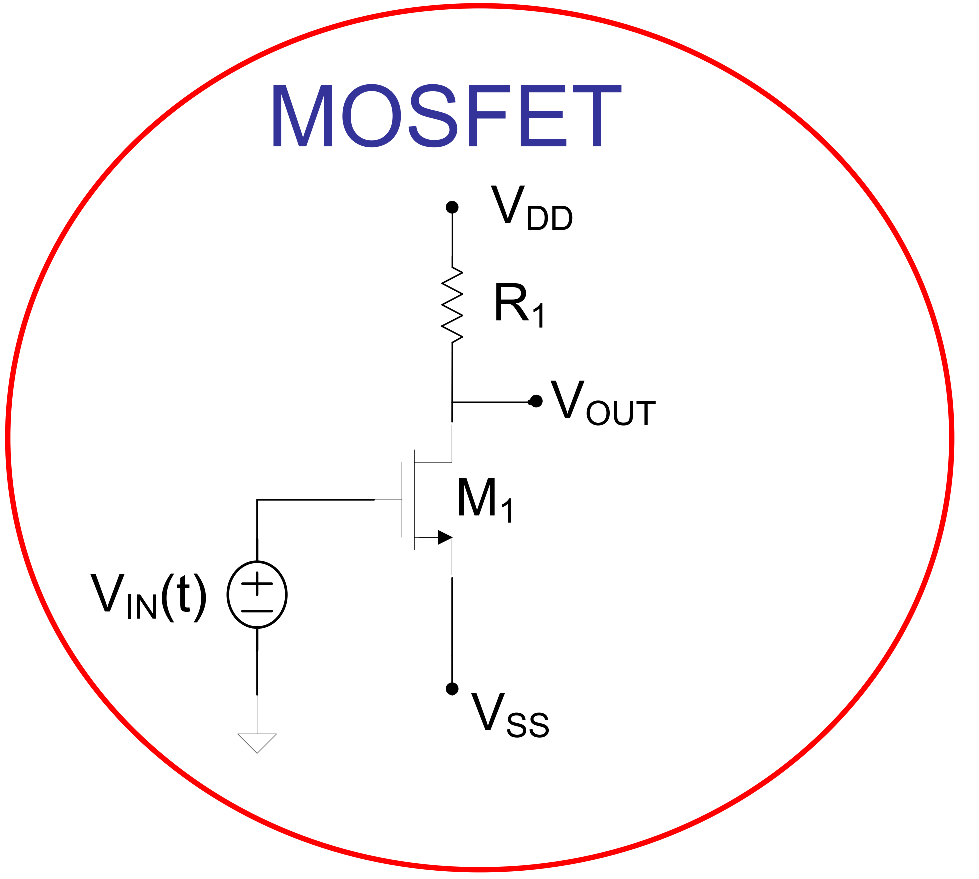
$$THD=\frac{\sqrt{(A_2 V_M)^2}}{|A_V V_M|}=\frac{A_2}{|A_V|}=\frac{\frac{\mu C_{OX} W}{4L} R V_M}{\frac{\mu C_{OX} W}{L} R(V_{GSQ}-V_T)}=\frac{V_M}{4(V_{GSQ}-V_T)}$$

Distortion will be small for $V_M \ll (V_{GSQ}-V_T)$

Distortion will be much worse (larger and more harmonic terms) if M_1 leaves saturation region.

Consider the following MOSFET and BJT Circuits



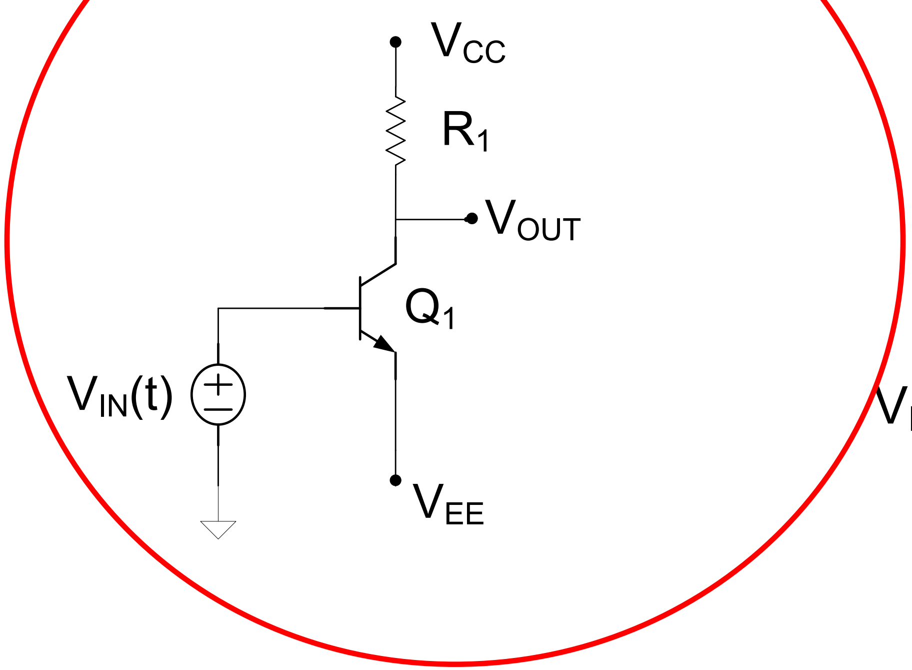


- Analysis was very time consuming
- Issue of operation of circuit was obscured in the details of the analysis

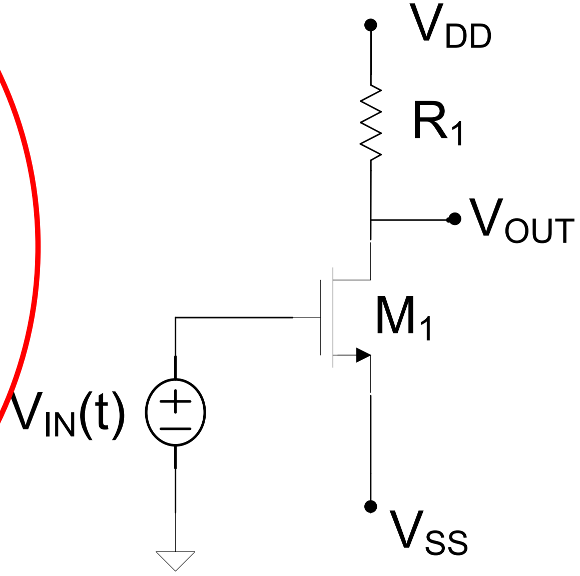
One of the most widely used amplifier architectures

Consider the following MOSFET and BJT Circuits

BJT



MOSFET



One of the most widely used amplifier architectures

Stay Safe and Stay Healthy !

End of Lecture 23